Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

TCS 455: Solution for Problem Set 5

Semester/Year: **2/2009**
Course Title: Mobile Communications
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Course Web Site: http://www.siit.tu.ac.th/prapun/ecs455/

Due date: Feb 5, 2010 (Friday)

1. Fourier transform review
   a. Give a ***simplified*** expression for the Fourier transform $C(f)$ of a waveform $c(t)$ when

$$c(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$

Solution

$$C(f) = \int_{-\infty}^{\infty} c(t) e^{-j2\pi ft} dt = \int_0^T A e^{-j2\pi ft} dt = A \frac{1}{-j2\pi f} e^{-j2\pi ft} \Big|_0^T$$

$$= A \frac{1}{-j2\pi f}\left(e^{-j2\pi fT} - 1\right) = \frac{A}{j2\pi f}\left(1 - e^{-j2\pi fT}\right)$$

   b. A message $m = (m_0, m_1, m_2, m_3) = (1, -1, 1, 1)$ is sent via

$$s(t) = \sum_{k=0}^{\ell-1} m_k c(t - kT) \text{ where } \ell \text{ is the length of } m.$$

Find a ***simplified*** expression for the Fourier transform $S(f)$ of the waveform $s(t)$.

Solution

We start with $s(t) = \sum_{k=0}^{\ell-1} m_k c(t-kT) \xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{\ell-1} m_k e^{-j2\pi fkT}$.

Hence,

$$
\begin{aligned}
S(f) &= C(f)\left(m_0 + m_1 e^{-j1\pi fT} + m_2 e^{-j2\pi fkT} + m_3 e^{-j2\pi fkT}\right); \ell = 4 \\
&= \frac{A}{j2\pi f}(1-z)\left(m_0 + m_1 z + m_2 z^2 + m_3 z^3\right);\ z = e^{-j1\pi fT} \\
&= \frac{A}{j2\pi f}(1-z)\left(1 - z + z^2 + z^3\right) \\
&= \frac{A}{j2\pi f}\left(1 - 2z - 2z^2 - z^4\right) \\
&= \frac{A}{j2\pi f}\left(1 - 2e^{-j1\pi fT} - 2e^{-j2\pi fT} - e^{-j4\pi fT}\right)
\end{aligned}
$$

c. Assume T = 2 [ms] and A = 1 [mV]. Find $S(0)$ and **plot** $|S(f)|$ where $f \in [-2, 2]$ kHz for the following *m*.

i. $m = (1)$

ii. $m = (1,1)$

iii. $m = (1,1,0,0)$

iv. $m = (1,1,-1)$

v. $m = (1,1,-1,1)$

vi. $m = (1,1,-1,-1)$

vii. $m = (1,1,-1,1,-1,-1,1,1,1,-1,1,1)$

Solution

First, we find

$$C(0) = \int_{-\infty}^{\infty} c(t) e^{-j2\pi 0t} dt = \int_{-\infty}^{\infty} c(t) dt = AT$$

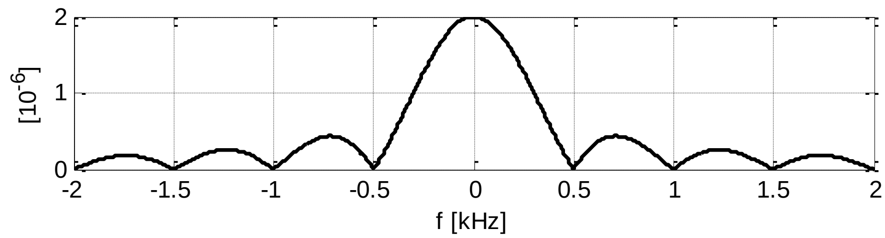
Then

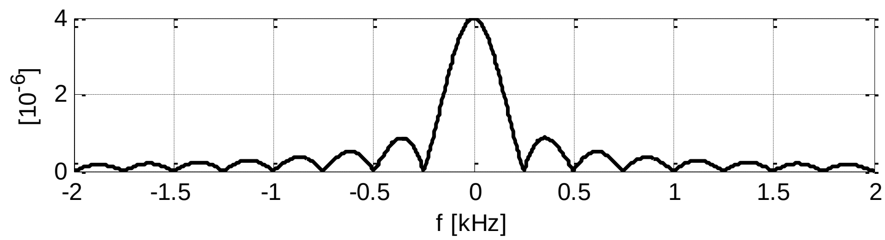
$$S(0) = C(0)\sum_{k=0}^{\ell-1} m_k e^{-j2\pi 0kT} = C(0)\sum_{k=0}^{\ell-1} m_k = AT\sum_{k=0}^{\ell-1} m_k$$

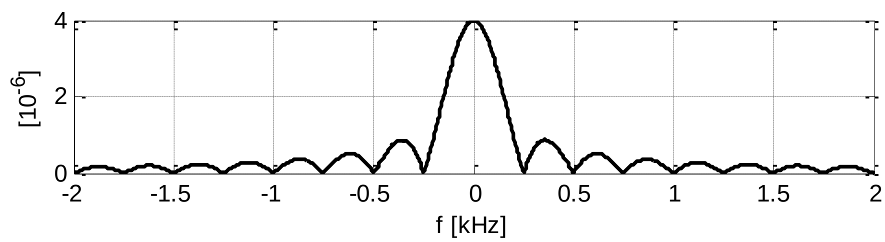
Plugging in the numbers, we have

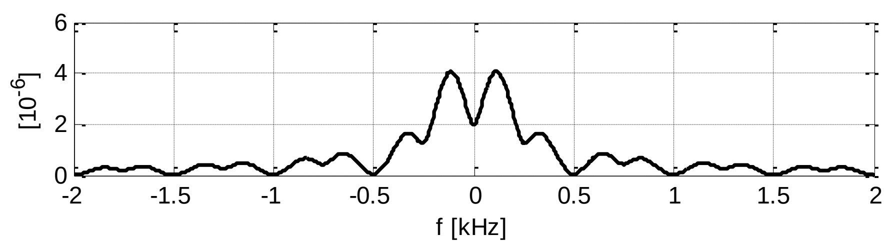
$$S(0) \;=\; 2,\ 4,\ 4,\ 2,\ 4,\ 0,\ 8 \qquad \times 10^{-6}$$

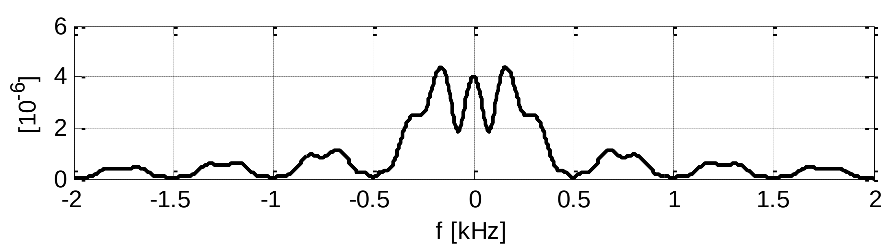
All the plots are shown on the next page.

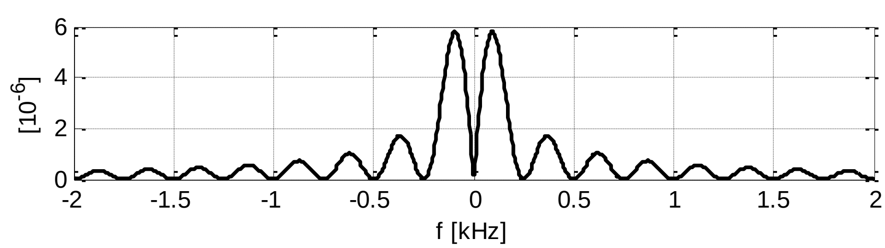

2
1
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]

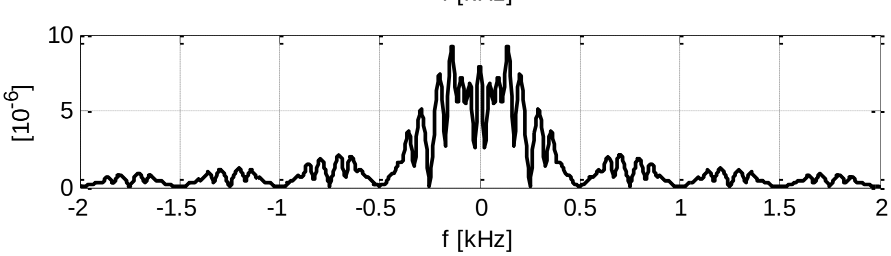

4
2
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]


4
2
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]


6
4
2
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]


6
4
2
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]


6
4
2
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]


10
5
0
$[10^{-6}]$
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
f [kHz]

2. Consider Global System for Mobile (GSM), which is a TDMA/FDD system that uses 25 MHz for the forward link, which is broken into radio channels of 200 kHz. If 8 speech channels are supported on a single radio channel, and if no guard band is assumed, find the number of simultaneous users that can be accommodated in GSM.

Solution

$$\frac{25\times 10^6}{200\times 10^3}\times 8 = 1000 \text{ simultaneous users.}$$

3. Draw the **<u>complete</u>** state diagrams for linear feedback shift registers (LFSRs) using the following polynomials. Does either LFSR generate an m-sequence?

Solution

(a) $1+x^2+x^5$	(b) $1+x+x^2+x^5$	(c) $1+x+x^2+x^4+x^5$
(a) The LFSR will cycle through the following states: 1 0 0 0 0 0 1 0 0 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 1 1 0 1 0 1 1 1 0 1 0 1 1 1 0 1 0 1 1 1 1 1 0 1 1 0 1 1 0 1 0 0 1 1 0 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 0 1 1 0 0 1 0 1 1 0 0 1 0 1 1 0 0 1 0 1 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0	(b) The LFSR will cycle through one of the cycles of states below. The initial state determine which cycle it will go through. Cycle #1: 10000 11000 01100 10110 11011 11101 11110 01111 00111 10011 01001 00100 00010 00001 Cycle #2: 00011 10001 01000 10100 11010 01101 00110 Cycle #3: 00101 10010 11001 11100 01110 10111 01011 00101	(c) The LFSR will cycle through the following states: 10000 11000 01100 10110 01011 10101 01010 00101 10010 01001 00100 00010 10001 01000 10100 11010 11101 11110 11111 01111 10111 11011 01101 00110 10011 11001 11100 01110 00111 00011 00001 10000

	Cycle #4: 0 1 0 1 0 1 0 1 0 1 Cycle #5: 1 1 1 1 1	

The polynomial $1+x^2+x^5$ and $1+x+x^2+x^4+x^5$ from part (a) and (c) generate m-sequences. (Their states go thorough cycle of size 2^5-1)

4. Use any resource, find all primitive polynomials of degree 6 over GF(2). Indicate your reference.

Solution

Primitive Polynomials
$x^6 + x^1 + 1$
$x^6 + x^5 + x^2 + x^1 + 1$
$x^6 + x^5 + x^3 + x^2 + 1$
$x^6 + x^4 + x^3 + x^1 + 1$
$x^6 + x^5 + x^4 + x^1 + 1$
$x^6 + x^5 + 1$

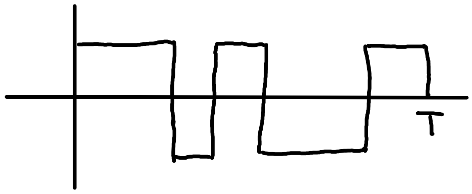
Source: http://www.theory.cs.uvic.ca/~cos/gen/poly.html

5.

HW5 Q5 Solution

Thursday, March 04, 2010
2:22 PM

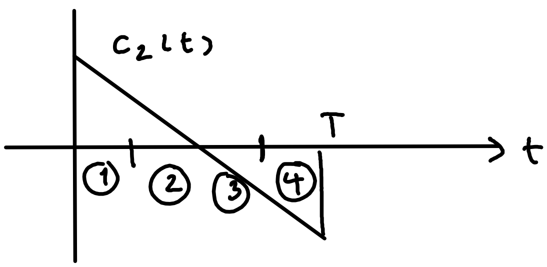
(Q5) We will try to find waveforms that simply oscillate between positive and negative value :



Now, to make it orthogonal to $c_1(t)$, the positive portion of the graph must be equal to the negative portion of the graph.

(So, the above waveform does not work.)

It should also be orthogonal to $c_2(t)$. One way to make this happen is to divide the interval $[0, T]$ into many parts, say 4 parts.

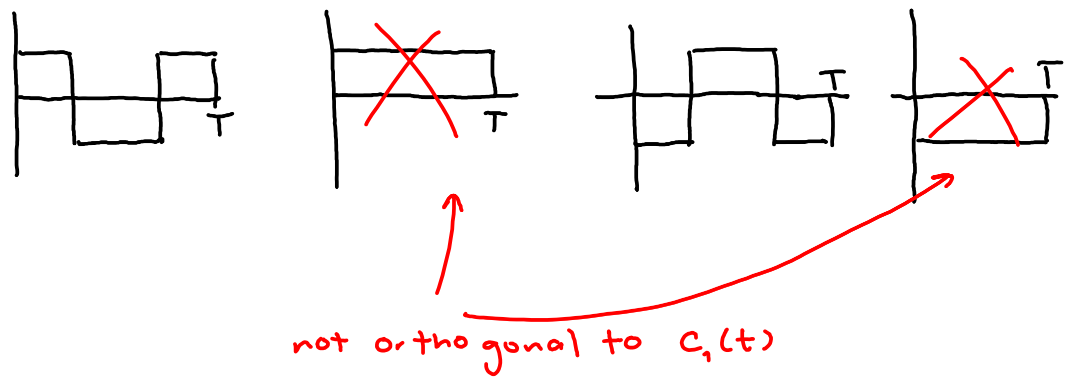


To ensure orthogonality, we must have sections ① and ④ multiplied by the same sign and sections ② and ③ multiplied by the same sign.

From this, there are 4 options

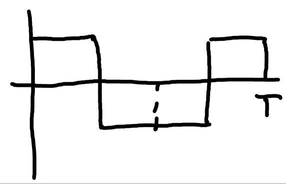
①, ④ × " >0 " ②, ③ × " >0 "

①, ④ × " <0 " ②, ③ × " <0 "



The two new waveforms are orthogonal to both $c_1(t)$ and $c_2(t)$ but they are not orthogonal to one another.

We will keep only one:

$c_3(t) =$ 

At this point, we have

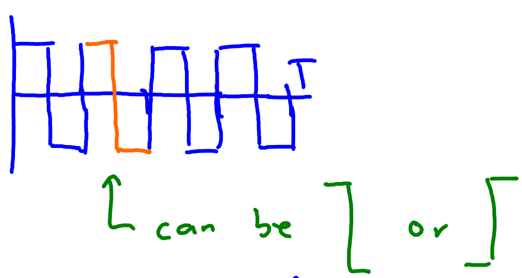
$$\int_0^T c_1(t)\, c_3(t)\, dt = \int_0^T c_2(t)\, c_3(t)\, dt = 0$$

To find $c_4(t)$,

we may have to further divide the interval

To do this,

an easy way is to split each section into positive and negative sub-section:



This way, the new waveform will be orthogonal to both $c_1(t)$ and $c_3(t)$.

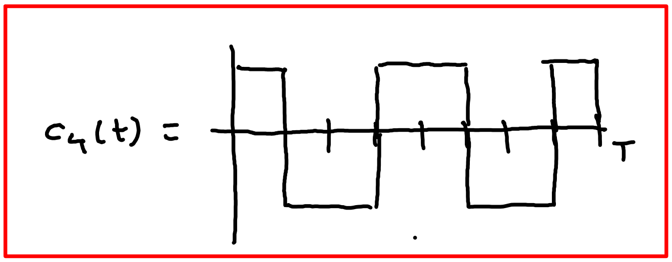
As drawn, it is not orthogonal to $c_2(t)$ so we will need to switch the sign.

Note that $c_2(t)$ is symmetric around $\frac{T}{2}$.

Hence, our $c_4(t)$ will be " " " as well;

Hence, our $c_4(t)$ will be " " " as well; so that their product will still have equal positive and negative areas.

One such option is:



The final step is to check that

$$\int_0^T c_1(t)c_2(t)\,dt = \int_0^T c_1(t)\,c_3(t)\,dt = \int_0^T c_1(t)\,c_4(t)\,dt = 0$$

$$\int_0^T c_2(t)\,c_3(t)\,dt = \int_0^T c_2(t)\,c_4(t)\,dt = \int_0^T c_3(t)\,c_4(t)\,dt = 0.$$

This is given to be $=0$.

The other integrations are $=0$ by construction.

6. In CDMA, each bit time is subdivided into *m* short intervals called **chips**. We will use 8 chips/bit for simplicity. Each station is assigned a unique 8-bit code called a **chip-sequence**. To transmit a 1 bit, a station sends its chip sequence. To transmit a 0 bit, it sends the one's complement[1] of its chip sequence.

   Here are the binary chip sequences for four stations:

   A: 0 0 0 1 1 0 1 1

   B: 0 0 1 0 1 1 1 0

   C: 0 1 0 1 1 1 0 0

   D: 0 1 0 0 0 0 1 0

   For pedagogical purposes, we will use a bipolar notation **with binary 0 being -1 and binary 1 being +1**. In which case, during each bit time, a station can transmit a 1 by sending its chip sequence, it can transmit a 0 by sending the negative of its chip sequence, or it can be silent and transmit nothing. We assume that all stations are synchronized in time, so all chip sequences begin at the same instant.

   When two or more stations transmit simultaneously, their bipolar signals add linearly.

   a. Suppose that A, B, and C are simultaneously transmitting 0 bits. What is the resulting (combined) bipolar chip sequence?

   b. Suppose the receiver gets the following chips: (-1 +1 -3 +1 -1 -3 +1 +1).
      Which stations transmitted, and which bits did each one send?

   Solution

```
%Chip sequences
C = [0 0 0 1 1 0 1 1; 0 0 1 0 1 1 1 0; 0 1 0 1 1 1 0 0; 0 1 0 0 0 0 1 0];
C = 2*C-1; %Change to bipolar form

% Part a
m = [-1 -1 -1 0] %message to transmit
x = %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Part b
r = [-1 1 -3 1 -1 -3 1 1] ;
m_decoded = 1/8* %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This gives [mA mB mC mD]' in bipolar form;
%The value is 1 if 1 was transmitted. The value is 0 if nothing was
%transmitted. The value is -1 if 0 was transmitted.
```

Use the above MATLAB code with x = m*C; and m_decoded = (C*r')/8;

(a) [3   1 1 -1 -3 -1 -1 1]

(b) [1 -1 0 1]'; Hence, A and D sent 1 bits, B sent a 0 bit, and C was silent.

[1] You should have seen the "one's complement" operation in your "digital circuits" class.